Tutorial 4 : LTL, CTL and Model Checking

CS60030 Formal Systems

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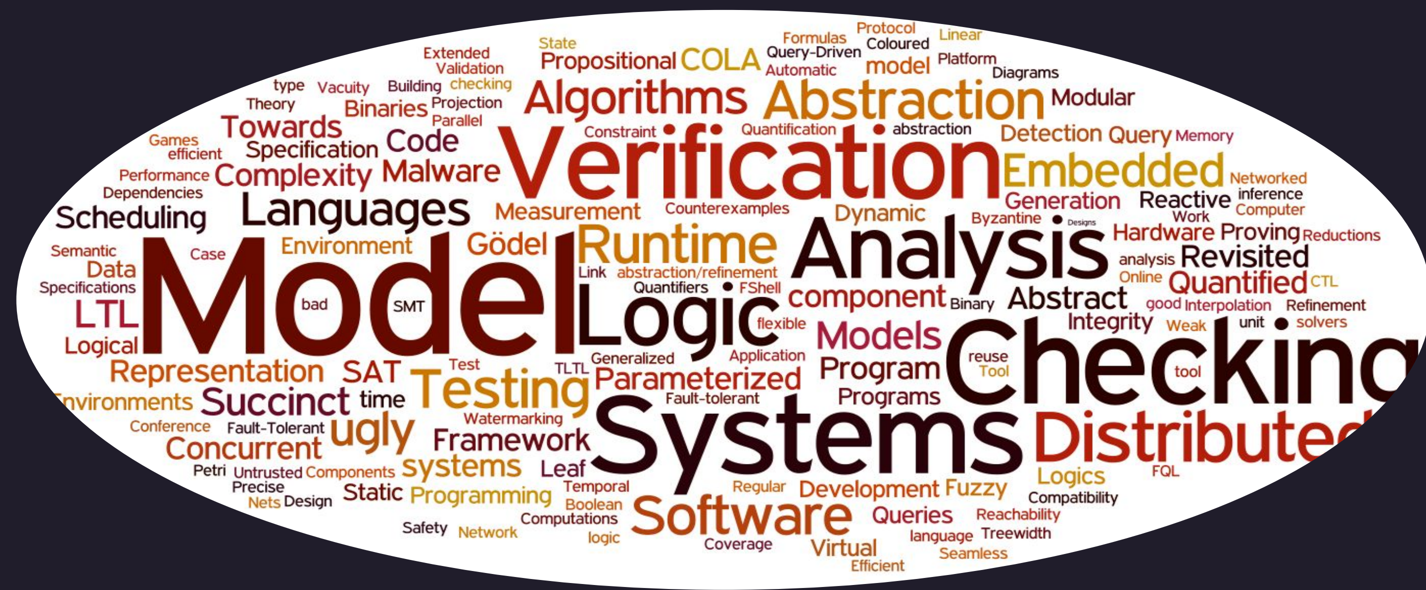


INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

FMSAFE

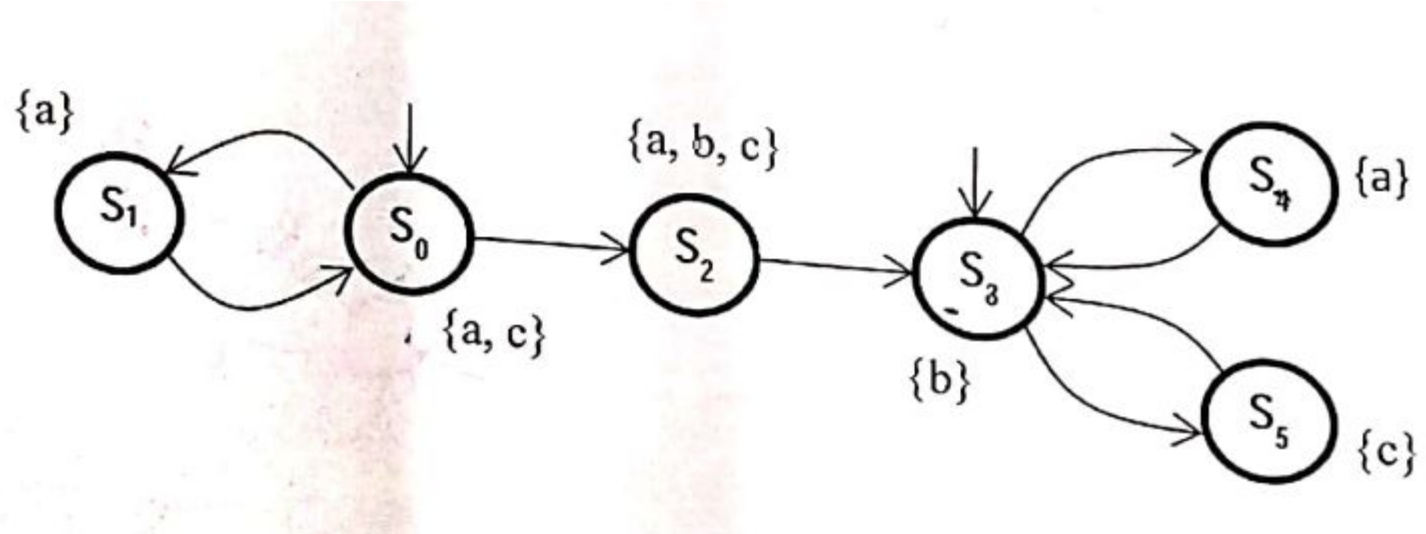
FORMAL METHODS FOR SAFETY CRITICAL SYSTEMS

LTL to GNBA

(a) Write the consistent elementary sets of sub-formulae of the LTL property: $(\neg X\ a) \vee (XX\ a)$

(b) Draw the non-deterministic Büchi automaton (NBA) for the LTL property of part (a). The sets represent the states of the NBA

LTL property writing



Decide for each LTL formula φi, whether TS |≠ φi. If TS |≠ φi then show a path π such that π |≠ φi .

1) $\varphi 1 = Fb$

2) $\varphi 2 = XX(c \lor b)$

3) $\varphi 3 = F(a \land b \land c)$

4) $\varphi 4 = (XXXa) \lor (FGa)$

5) $\varphi 5 = (a \lor b)\ U\ (a \lor c)$

6) $\varphi 6 = G\ (b \Rightarrow XFc)$

7) $\varphi 7 = F\ (c \land (a \lor b))$

LTL property checking

Consider the set AP of atomic propositions defined by AP = { x=0, x>1 } and consider a non-terminating process that manipulates the variable x. Formulate the following informally stated properties as LTL properties. Sometimes you may have to write whatever is implied in terms of the atomic propositions.

(i) False

(ii) Initially x is equal to zero

(iii) Initially x differs from zero

(iv) Initially x is equal to zero, but at some point x exceeds one

(v) x exceeds one only finitely many times

(vi) x exceeds one infinitely often

(vii) The value of x alternates between zero and two

(viii) True

LTL Properties

c : Class

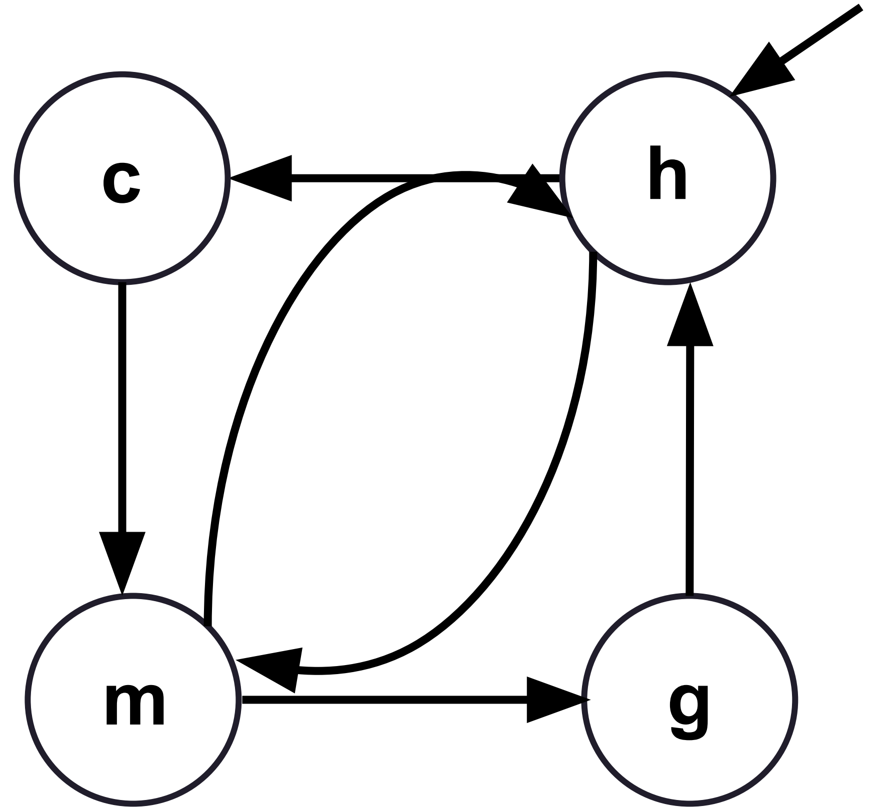
h : Hostel

m : Mess

g : Gymkhana

Write the LTL formulations for the following sentences:

1. The Mess is visited infinitely often
2. Eventually the class is always visited.
3. Once in class a student eventually goes to Mess after spending some time is Gymkhana



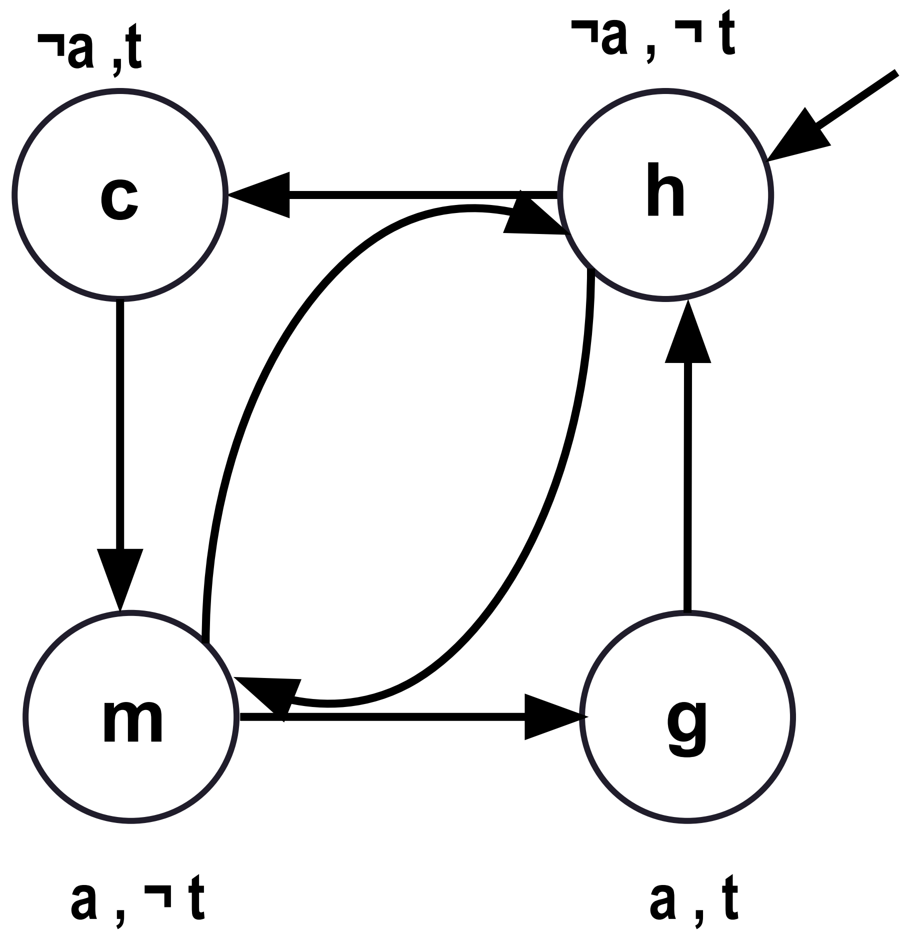
CTL Properties

a : attention is high

t : tempo is high

Write the CTL formulations for the following sentences and check on the given model:

1. There exists a path where in future on all paths always ¬a
2. There exists a path where always ¬t
3. There exists a path where a does not hold till t
4. On all paths always a holds or there exists a path where always ¬t holds
5. EG (a ⋁ EG ¬t)
6. AGAF (a ⋁ ¬t)



LTL Properties

Write the LTL formulations for the following sentences:

1. A and B always alternate starting with A. This means only A is true in the first step, then only B is true in the next step, and this alternation between A and B is always repeated.
2. Two neighboring A's never occur
3. Between two neighboring A's there is at least one B.
4. Never is it that an A is followed by a B unless the A is preceded by a C
5. If at some point C holds and at all points before it A did not hold and B held, then at some point after C, A and B both hold.

CTL Properties

Write the CTL formulations for the following sentences:

S : I follow Social distancing

V : I got Vaccinated

1. I will follow social distancing, no matter what happens.
2. It's possible I may be follow social distancing some day, at least for one day.
3. It's always possible that I will suddenly be following social distancing for the rest of time.
4. Depending on what happens in the future, it's possible that for the rest of time, I'll be guaranteed at least one day of following social distancing still ahead of me.
5. There will surely be a time in future when I get vaccinated. Until then I have to follow social distancing.
6. From now on until I get vaccinated, I will follow social distancing. Once I get vaccinated, I may not follow social distancing.

Equivalence checking

Check if the following LTL properties are equivalent :

1. $G(A \rightarrow B) \equiv (GA \rightarrow GB)$
2. $GFp \rightarrow GFq \equiv G(p \rightarrow Fq)$
3. $FGp \wedge FGq \equiv FG(p \wedge q)$
4. $pU(qUr) \equiv r \vee ((p \vee q) \wedge (p\,U\,(q\,U\,r))$

LTL to GNBA

(a) Write the consistent elementary sets of sub-formulae of the LTL property: $(a \wedge \neg b) \, U \, (\neg a \, U \, b)$

(b) Draw the non-deterministic Büchi automaton (NBA) for the LTL property of part (a). The sets represent the states of the NBA

LTL to GNBA

elementary sets

$B_1 \equiv \{a, b, \neg\varphi_1, \varphi_2, \varphi\}$ I F_1 F_2

$B_2 \equiv \{a, \neg b, \varphi_1, \neg\varphi_2, \varphi\}$ I F_2

$B_3 \equiv \{a, \neg b, \varphi_1, \neg\varphi_2, \neg\varphi\}$ F_1 F_2

B_5 $B_4 \equiv \{\neg a, \neg b, \neg\varphi_1, \varphi_2, \varphi\}$ I F_1

B_6 $B_5 \equiv \{\neg a, \neg b, \neg\varphi_1, \neg\varphi_2, \neg\varphi\}$ F_1 F_2

B_4 $B_6 \equiv \{\neg a, b, \neg\varphi_1, \varphi_2, \varphi\}$ I F_1 F_2

$\phi_1 \cup \phi_2 \equiv \phi_2 \vee (\phi_1 \wedge \bigcirc \phi_2)$

$\neg(\phi_1 \cup \phi_2) \equiv \neg\phi_2 \wedge (\neg\phi_1 \vee \bigcirc \neg\phi_2)$

B_5 B_6 B_4

$F_1 = \{B_1, B_3, B_4, B_5, B_6\}$ for φ

$F_2 = \{B_1, B_2, B_3, B_5, B_6\}$ for φ_2

B_6 B_4